Light scattering by an infinite circular cylinder immersed in an absorbing medium

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Analytic solutions are developed for the single-scattering properties of an infinite dielectric cylinder embedded in an absorbing medium with normal incidence, which include extinction, scattering and absorption efficiencies, the scattering phase function, and the asymmetry factor. The extinction and scattering efficiencies are derived by the near-field solutions at the surface of the particle. The normalized scattering phase function is obtained by use of the far-field approximation. Computational results show that, although the absorbing medium significantly reduces the scattering efficiency, it has little effect on absorption efficiency. The absorbing medium can significantly change the conventional phase function. The absorbing medium also strongly affects the polarization of the scattered light. However, for large absorbing particles the degrees of polarization change little with the medium's absorption. This implies that, if the transmitting lights are strongly weakened inside the particle, the scattered polarized lights can be used to identify objects even when the absorption property of the host medium is unknown, which is important for both active and passive remote sensing. © 2005 Optical Society of America

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1. Introduction

Light scattering by a spherical particle immersed in an absorbing medium has been studied by several authors.[1–11] Actually, most particles in nature have nonspherical shapes that include cylinderlike particles in absorbing media. Light scattering by an infinite circular cylinder[12–14] or finite circular cylinder[15] in a nonabsorbing medium is well understood. Light extinction by a circular cylinder in an absorbing medium has also been studied recently.[16] However, to our knowledge a complete solution of light scattering by a cylinder in an absorbing medium has not been developed. In this study, analytic solutions are derived for the single-scattering properties of an infinite dielectric cylinder embedded in an absorbing medium with normal incidence. The extinction and scattering efficiencies are derived by near-field solutions at the surface of the particle. The normalized scattering phase function is calculated from the far-field approximation. Computational results are presented to address the absorption effects of the host medium on single-scattering properties of the infinite cylinder.

2. Theory

We consider an infinite dielectric circular cylinder with radius a and a refractive index m_t immersed in an absorbing medium with a complex refractive index m, which is illuminated by a normally incident plane wave propagating in the reverse x direction in a coordinate system as shown in Fig. 1. The central axis of the cylinder is assumed to be the z axis of the coordinate system. The incident electric field at the z axis is assumed to be E_0. There are two possible orthogonal polarization states of the incident wave: an electric field polarized parallel to the x, z plane and an electric field polarized perpendicular to the x, z plane.[17] Light-scattering properties of the particle can be derived directly for the two orthogonal polarization states of the incident wave. For each polarization state of the incidence, we expand the incident, scattered, and internal electromagnetic fields in vector cylindrical harmonics.[17] For example, when the incident electric field is parallel to the x, z plane, the incident fields ($\mathbf{E}_i$ and $\mathbf{H}_i$) and the scattered fields ($\mathbf{E}_s$ and $\mathbf{H}_s$) can be expressed as

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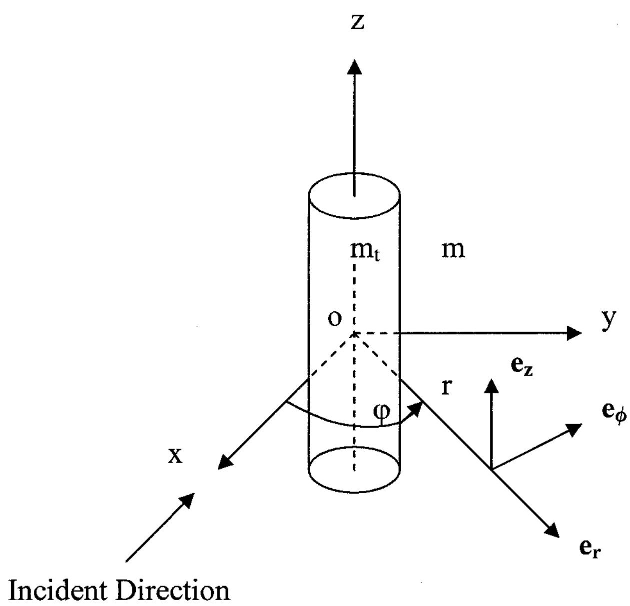

Fig. 1. Geometry of light scattering by a circular cylinder embedded in an absorbing medium.

$$\mathbf{E}_i = \sum_{n=-\infty}^{\infty} E_n \mathbf{N}_n^{(1)}, \tag{1a}$$

$$\mathbf{H}_i = -\frac{ik}{\omega\mu} \sum_{n=-\infty}^{\infty} E_n \mathbf{M}_n^{(1)}, \tag{1b}$$

$$\mathbf{E}_s = -\sum_{n=-\infty}^{\infty} E_n b_n \mathbf{N}_n^{(3)}, \tag{1c}$$

$$\mathbf{H}_s = \frac{ik}{\omega\mu} \sum_{n=-\infty}^{\infty} E_n b_n \mathbf{M}_n^{(3)}, \tag{1d}$$

where $E_n = (-i)^n E_0/k$, $i = \sqrt{-1}$, and $k = 2\pi m/\lambda_0$, where λ_0 is the wavelength in vacuum, μ is the permeability of the medium, and ω is the angular frequency of the wave. The vector cylindrical harmonics $\mathbf{M}_n$ and $\mathbf{N}_n$ are given by

$$\mathbf{M}_n = k \exp(in\varphi)\left[in\frac{z_n(\rho)}{\rho}\, \mathbf{e}_r - z_n{}'(\rho)\mathbf{e}_\varphi \right], \tag{2a}$$

$$\mathbf{N}_n = k \exp(in\varphi) z_n(\rho)\mathbf{e}, \tag{2b}$$

where $\rho = kr$ and $z_n(\rho)$ denotes the Bessel functions. The prime superscript in Eq. (2a) denotes the differential of $z_n(\rho)$. Superscripts to $\mathbf{M}_n$ and $\mathbf{N}_n$ in Eqs. (1a)–(1d) denote the kind of Bessel function $z_n(\rho)$: (1) denotes the Bessel function of the first kind $J_n(\rho)$ and (3) denotes the Hankel function $H_n^{(1)}(\rho) = J_n(\rho) + iY_n(\rho)$, where $Y_n(\rho)$ is the Bessel function of the second kind.

The scattering coefficient b_n in Eqs. (1c) and (1d) is derived by use of the expansions of the fields and continuity conditions on the boundary of the cylinder as[17]

$$b_n = \frac{mJ_n(m_t x)J_n{}'(mx) - m_t J_n{}'(m_t x)J_n(mx)}{mJ_n(m_t x)H_n^{(1)'}(mx) - m_t J_n{}'(m_t x)H_n^{(1)}(mx)}, \tag{3}$$

where $x = 2\pi a/\lambda_0$. Since $J_{-n} = (-1)^n J_n$ and $Y_{-n} = (-1)^n Y_n$, we know from Eq. (3) that $b_{-n} = b_n$.

The rates of energy scattered and attenuated by a unit length of the cylinder are

$$\begin{aligned} W_s &= \frac{1}{2}\mathrm{Re}\left[\int_0^{2\pi} (\mathbf{E}_s \times \mathbf{H}_s{}^*)_r a \mathrm{d}\varphi \right] \\ &= -2aI_0\, \mathrm{Im}\left\{ \pi\left(1 - i\frac{m_i}{m_r}\right)\left[b_0 b_0{}^* H_0^{(1)}(mx) H_0^{(1)'*}(mx) \right.\right. \\ &\quad \left.\left. + 2\sum_{n=1}^{\infty} b_n b_n{}^* H_n^{(1)}(mx) H_n^{(1)'*}(mx) \right] \right\}, \end{aligned} \tag{4a}$$

$$\begin{aligned} W_e &= \frac{1}{2}\,\mathrm{Re}\left[\int_0^{2\pi} (\mathbf{E}_i \times \mathbf{H}_i{}^* + \mathbf{E}_i \times \mathbf{H}_s{}^* + \mathbf{E}_s \right. \\ &\quad \left. \times \mathbf{H}_i{}^*)_r a \mathrm{d}\varphi \right] \\ &= 2aI_0\, \mathrm{Im}\left\{ \pi\left(1 - i\,\frac{m_i}{m_r}\right)\left[J_0(mx)J_0{}'^*(mx) \right.\right. \\ &\quad - b_0{}^* J_0(mx) H_0^{(1)'*}(mx) - b_0 J_0{}'^*(mx) H_0^{(1)}(mx) \\ &\quad + 2\sum_{n=1}^{\infty} \big(J_n(mx)J_n{}'^*(mx) - b_n{}^* J_n(mx) H_n^{(1)'*}(mx) \\ &\quad \left.\left. - b_n J_n{}'^*(mx) H_n^{(1)}(mx) \big) \right] \right\}, \end{aligned} \tag{4b}$$

where $I_0 = 1/2(m_r/c\mu)|E_0|^2$, c is the speed of light in vacuum, and m_i and m_r are the imaginary and real parts of the host refractive index m. The subscript r in Eqs. (4a) and (4b) denotes the vector component in the radial direction as shown in Fig. 1.

Similar to the solutions for the parallel polarized incident light, when the incident electric field is perpendicular to the x, z plane, the incident and scattered fields can be expressed as

$$\mathbf{E}_i = -i \sum_{n=-\infty}^{\infty} E_n \mathbf{M}_n^{(1)}, \tag{5a}$$

$$\mathbf{H}_i = -\frac{ik}{\omega\mu} \sum_{n=-\infty}^{\infty} E_n \mathbf{N}_n^{(1)}, \tag{5b}$$

$$\mathbf{E}_s = i \sum_{n=-\infty}^{\infty} E_n a_n \mathbf{M}_n^{(3)}, \tag{5c}$$

$$\mathbf{H}_s = \frac{ik}{\omega\mu} \sum_{n=-\infty}^{\infty} E_n a_n \mathbf{N}_n^{(3)}. \tag{5d}$$

The scattering coefficient a_n in Eqs. (5c) and (5d) is

$$a_n = \frac{m_t J_n(m_t x)J_n{}'(mx) - mJ_n{}'(m_t x)J_n(mx)}{m_t J_n(m_t x)H_n^{(1)'}(mx) - mJ_n{}'(m_t x)H_n^{(1)}(mx)}. \tag{6}$$

Note here that since $J_{-n} = (-1)^n J_n$ and $Y_{-n} = (-1)^n Y_n$, we also have $a_{-n} = a_n$.

The rates of energy scattered and attenuated by a unit length of the cylinder, respectively, are

$$W_s = \frac{1}{2}\mathrm{Re}\left[\int_0^{2\pi} (\mathbf{E}_s \times \mathbf{H}_s{}^*)_r a \mathrm{d}\varphi\right]$$
$$= 2aI_0 \,\mathrm{Im}\left\{\pi\left(1 - i\frac{m_i}{m_r}\right)\left[a_0 a_0{}^* H_0^{(1)'}(mx) H_0^{(1)*}(mx) + 2\sum_{n=1}^{\infty} a_n a_n{}^* H_n^{(1)'}(mx) H_n^{(1)*}(mx)\right]\right\}, \tag{7a}$$

$$W_e = \frac{1}{2}\,\mathrm{Re}\left[\int_0^{2\pi} (\mathbf{E}_i \times \mathbf{H}_i{}^* + \mathbf{E}_i \times \mathbf{H}_s{}^* + \mathbf{E}_s \times \mathbf{H}_i{}^*)_r a \mathrm{d}\varphi\right]$$
$$= -2aI_0 \,\mathrm{Im}\left\{\pi\left(1 - i\frac{m_i}{m_r}\right)\left[J_0{}'(mx) J_0{}^*(mx) - a_0{}^* J_0{}'(mx) H_0^{(1)*}(mx) - a_0 J_0{}^*(mx) H_0^{(1)'}(mx) + 2\sum_{n=1}^{\infty} \left(J_n{}'(mx) J_n{}^*(mx) - a_n{}^* J_n{}'(mx) H_n^{(1)*}(mx) - a_n J_n{}^*(mx) H_n^{(1)'}(mx)\right)\right]\right\}. \tag{7b}$$

The rate of energy incident on a unit length of cylinder can be numerically calculated from

$$f = 2aI_0 \int_0^{\pi/2} \exp(2m_i x \cos\alpha)\cos\alpha \mathrm{d}\alpha. \tag{8}$$

Therefore for each polarization state of the incidence, extinction, scattering, and absorption efficiencies are $Q_e = W_e/f$, $Q_s = W_s/f$, and $Q_a = (W_e - W_s)/f$, respectively. The unpolarized extinction, scattering, and absorption efficiencies are simply the averages of the quantities in the two polarization states.

The scattering phase-matrix and asymmetry factor are calculated from the elements of the amplitude scattering matrix. For a circular cylinder in an absorbing medium, the nonzero elements of the amplitude scattering matrix can be derived from the far-field approximation[17]:

$$s_1(\theta) = b_0 + 2\sum_{n=1}^{\infty} b_n \cos(n\theta), \tag{9a}$$

$$s_2(\theta) = a_0 + 2\sum_{n=1}^{\infty} a_n \cos(n\theta), \tag{9b}$$

where $\theta = \pi - \varphi$, which is the scattering angle. With the amplitude scattering matrix, it is straightforward to calculate all the elements of the Mueller matrix.[17] It should be emphasized that when the host medium is absorbing, the Mueller matrix is dependent on the distance from the cylinder. In the far-field zone (where the damping of the scattered rate follows $\exp[(4\pi m_i/\lambda_0)(r - a)]$ and r is the radius from the central axis of the cylinder) the normalized phase matrix is from the Mueller matrix normalized by the local scattered rate from a unit length of the cylinder, which is an integral of the first element of the Mueller matrix $\{s_{11}(\theta) = (1/2)[|s_1(\theta)|^2 + |s_2(\theta)|^2]\}$ around the cylinder.

3. Results

On the basis of the theory in Section 2, a computer program is developed for the scattering and absorption of electromagnetic waves by a cylinder in an absorbing medium. The computational results from this program for cylinders in both nonabsorbing and absorbing media are presented in this section. For light scattering by a cylinder in a nonabsorbing medium, the results from this program are the same as those from the code developed by Bohren and Huffman.[17]

As an example, we assume a cylinder with a refractive index of $m_t = 1.4 + i0.05$ embedded in a medium with a refractive index of $m = 1.2 + im_i$, where $m_i = 0.0, 0.001, 0.01$, and 0.05. The refractive indices applied here are those used in Mundy *et al.*[1] Various imaginary refractive indices of the host medium are selected to examine the effects of the absorbing medium on the absorption and scattering of light by the cylinder.

Figure 2 shows the single-scattering properties of a cylinder embedded in the assumed host medium as functions of the size parameter in free space ($2\pi a/\lambda_0$), which include extinction, scattering and absorption efficiencies, and the asymmetry factor. We can see that the absorbing medium has little effect on the absorption efficiency. For small cylinders, the asymmetry factor does not show a big change with the increase of absorption in the host medium until the size parameter of $\sim$10. However, for large cylinders the asymmetry factor is significantly reduced by the absorbing medium and can be negative when the absorption and size parameter are both large enough. There are three reasons for this effect: (1) when an absorbing particle is large, the light transmitting through it is weak, which reduces the forward scattering; (2) when the host medium's absorption is strong, the diffracted light experiences larger damping and results in weaker forward scattering; and (3) the incident light undergoes less damping on the side facing the incidence. This is similar to the cases for large spherical particles.[6,7] The absorbing medium also reduces the scattering efficiency. When the medium's absorption is increased, the scattered lights becomes weaker, especially when the size parameter is larger than $\sim$2. The asymptotic limits of the extinction, scattering, and absorption efficiencies on the size parameter for the cylinder embedded in the absorbing medium should be identical to those for

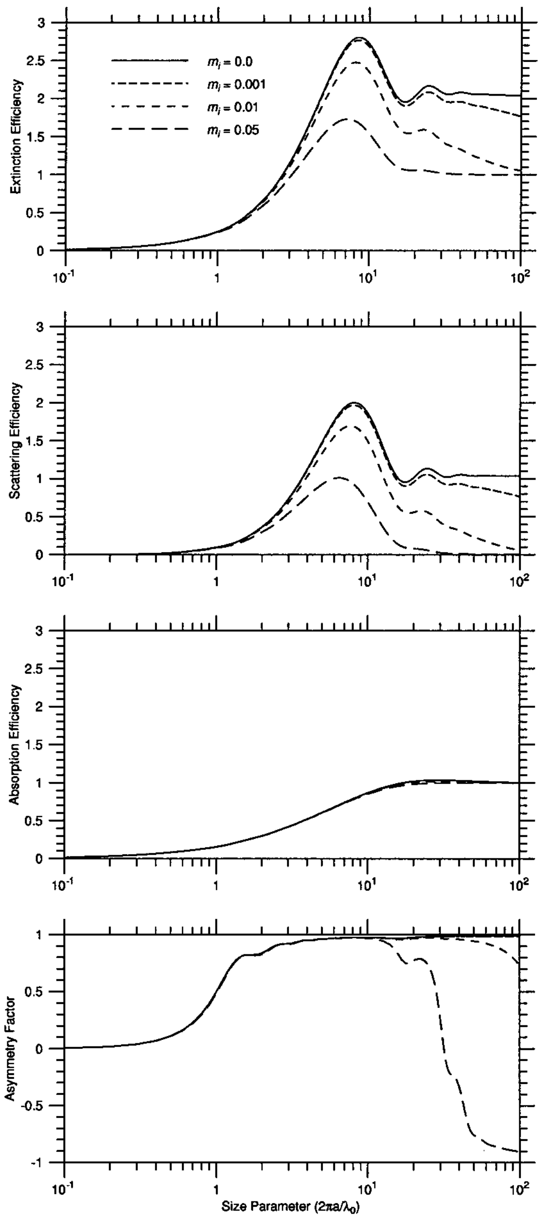

Fig. 2. Single-scattering properties of a cylinder embedded in a medium as functions of the size parameter in free space $(2\pi a/\lambda_0)$, which include extinction, scattering and absorption efficiencies, and the asymmetry factor. The refractive index of the cylinder is $m_t = 1.4 + i0.05$ and the refractive index of the medium is $m = 1.2 + im_i$, where $m_i = 0.0$, 0.001, 0.01, and 0.05, respectively.

spherical particles given in Sudiarta and Chylek[9] because, when a cylinder is infinitely large, we can calculate its reflection by considering it as a material half-space facing the incidence. To this limit, there is no difference in the geometrical-optics formulism of a sphere and cylinder. Comparing the results for spherical particles in Fig. 3 of Fu and Sun,[7] we find that the absorbing medium tends to affect the single-scattering properties of cylinders and spheres in a similar way although the single-scattering properties of the two particle shapes are significantly different. Therefore the effect of the absorbing medium on single-scattering properties of particles with different shapes should hold a similar trend.

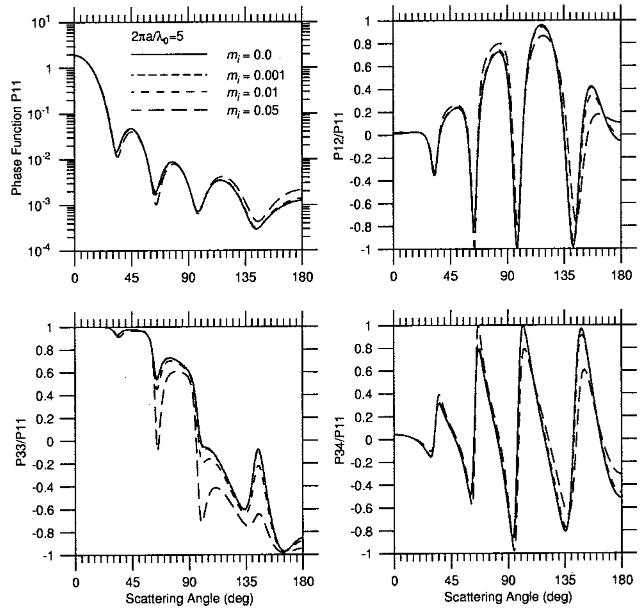

Fig. 3. Normalized nonzero phase-matrix elements of a cylinder embedded in a medium as functions of scattering angle. The refractive index of the cylinder is $m_t = 1.4 + i0.05$ and the refractive index of the medium is $m = 1.2 + im_i$, where $m_i = 0.0$, 0.001, 0.01, and 0.05, respectively. The size parameter of the cylinder in free space $(2\pi a/\lambda_0)$ is 5.

The normalized nonzero phase-matrix elements as functions of the scattering angle are shown in Figs. 3–5 for size parameters of 5, 25, and 100, respectively. We can see that the absorbing medium can significantly change the conventional phase functions when the size parameters are larger than 5. The

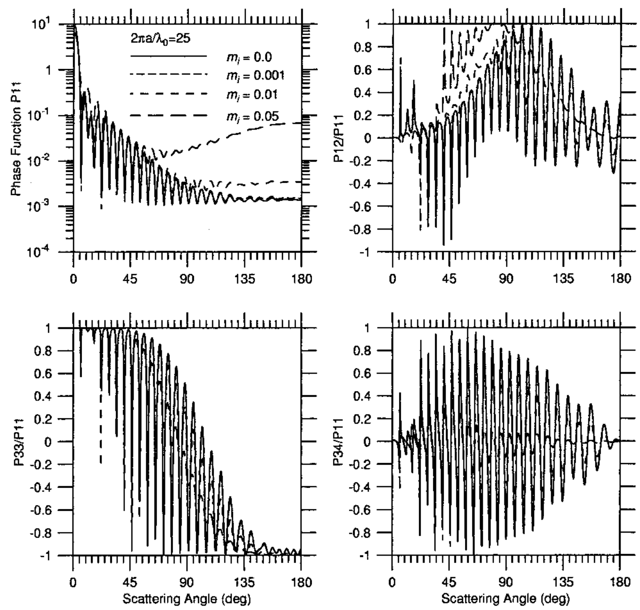

Fig. 4. Same as Fig. 3, but the size parameter of the cylinder in free space $(2\pi a/\lambda_0)$ is 25.

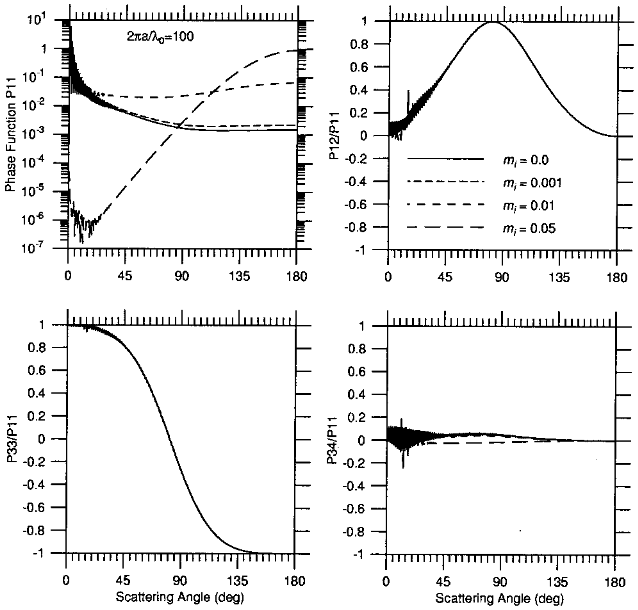

Fig. 5. Same as Fig. 3, but the size parameter of the cylinder in free space ($2\pi a/\lambda_0$) is 100.

absorbing medium also strongly affects the polarization degrees of the scattered light (i.e., $P12/P11$, $P33/P11$, and $P34/P11$) for the intermediate size parameter of 25. However, when the size parameter is 100, the degrees of polarization change little with the medium's absorption, and for larger size parameters the degrees of polarization are very similar to this result. A plausible explanation for this result is that for large absorbing particles the transmitting lights inside the particle are strongly weakened before they emerge as scattered light. The scattered lights are dominated by waves simply reflected from the particle surface. The waves of different polarization states that experience only the reflection process must undergo identical propagation paths in the absorbing host medium. Therefore the damping of the waves of different polarization states should also be identical, and this results in unchanged polarization degrees.

4. Conclusions

Analytic solutions are developed for the single-scattering properties of an infinite dielectric cylinder embedded in an absorbing medium with normal incidence, which include extinction, scattering and absorption efficiencies, the scattering phase function, and the asymmetry factor. Computational results show that, although the absorbing medium significantly reduces the scattering efficiency, it has little effect on the absorption efficiency. The absorbing medium can significantly change the conventional phase function. The absorbing medium also strongly affects the polarization degrees of the scattered light. However, for large absorbing particles the degrees of polarization change little with the medium's absorption. This implies that, if the transmitting lights are strongly weakened inside the particle, the scattered polarized lights can be used to identify objects even when the absorption property of the host medium is unknown, which is important for both active and passive remote sensing.

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